Quantifying Adaptability

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Abstract—**Until adaptability can be calculated and measured, it is difficult to understand or promote. This paper defines adaptability and offers a way to measure it based on communications theory. Adaptability is a complex process which requires a number of supporting processes and definitions. The concept of state-pairs is developed and used to describe communicating entities, interfaces, comparison, measured information, communications, flexibility and, finally, adaptability. Interfaces are delineated by their percent of adaptability; different means are identified to add the benefits of adaptability to existing and future systems.**

Keywords-adaptability; interface; communications structure; measured information; etiquette

I. Introduction

This paper defines *adaptability* as the process of automatically negotiating possible parameters, as it makes a system more adaptable. Some assumptions in this definition include: a negotiation occurs between at least two entities, and a negotiation requires communications. With the recognition that communications is a requirement before adaptability can be realized, adaptability is defined based on communications theory. First the structure of a communications system used to transfer a message between transmitter and receiver entities is developed. From this structure an interface is derived. Defining an interface allows an understanding of how to adapt and measure an interface. Quantifying adaptability also adds to the understanding of how to implement adaptable systems.

II. A Communications Structure

Adaptability requires communications. Fig. 1 models communications for the purpose of understanding its structure rather than its performance. Fig. 1 is similar to the Shannon model of a communications system except that the communications channel is replaced by an interface and the probability of the output message being the same as the input message is fixed to one [1, page 34]. The transmitter (T) and receiver (R) are communicating entities connected via an interface. This model allows an analysis of the structure of the relationship between T and R.

From communications theory, T and R support all the state-pairs t_i - r_i, where i includes the set of all t or r states 1 to n in Fig. 1. A state-pair includes a specific input part (t_x) associated with T, which is related to the output part (r_x) associated with R.

The relationship between t_x and r_x is described as one-one. "A relation is said to be one-one when, if t has the relation in question to r, no other term t' has the same relation to r, and t does not have the same relation to any other term r' other than r" [2]. All the state-pairs associated with T and R form the interface between T and R. A single set of t_i or r_i states is termed a parameter of the transmitter or receiver. Communications between T and R (information transfer) is possible only when multiple state-pairs form an interface. Most interfaces include multiple parameters. An interface is a concept that does not exist independently, it is only formed by the common parameters of the related entities.

A state-pair is in some ways a relationship between equal elements. As example in Fig. 1, state t_x may be considered as the equal of state r_x. However, it is not possible to define such equality exactly without specifying other sets of state-pairs. For state t_x to be equal to r_x for example, the boundary conditions of each state (defined by other state-pairs) must be equal. To avoid this complexity, this paper considers a state-pair to be one-one, not necessarily equals.

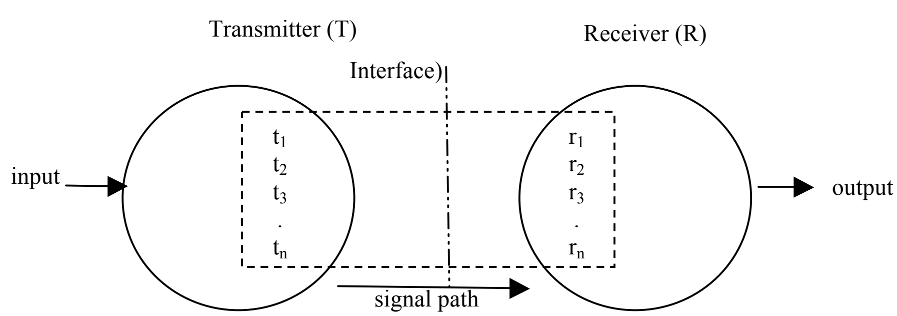

Figure 1. Communications structure.

The concept of state-pairs may be applied to any interface, even a physical interface. Examine a perfectly compatible (zero tolerance) physical plug and socket. The outside diameter of the plug and the inside diameter of the socket are the same. The length of the plug and the socket are the same. The interface between the plug and socket consists of all the points on the same diameters along the same length of the socket and plug. These common points are the state-pairs which form a physically compatible interface. In a real physical interface, multiple layers of sets of state-pairs are needed to completely define the interface including: the physical dimension system used and the tolerances applied.

III. COMMUNICATIONS PROCESS

The communications structure in Fig. 1 allows a detailed view of a communications process across the interface. This is necessary to see the how state-pairs create an interface and eventually to use the number of state-pairs to quantify adaptability.

The ability to pass information across each state-pair requires two comparisons. Each comparison is associated with a part of a state-pair. The fundamental nature of these comparisons is suggested by I. Kant who states that a comparison is necessary for understanding [3].

TABLE I. COMMUNICATIONS PROCESS

For the transmitter (T):	
1	Select an input message
2	Compare this input and determine state (t_i)
3	Output a signal
For the receiver (R):	
4	Select the signal received
5	Compare this signal and determine related state (r_i)
6	Output message

The simplest communications process may consist of six operations, three in the transmitter and three in the receiver (Table 1). What is critical to this analysis is that the communications process consists of symmetric transmit and receive processes, each of which includes a comparison. The symmetry about the interface first appears in the concept of one-one and also appears in Fig. 3 (below). Operations 2 and 5 in Table 1 demonstrate how a state-pair relates to the communications process.

Consider a binary amplitude-modulated communications system with two state-pairs (t_1 - r_1 and t_2 - r_2) and without time domain or tolerance effects. The input message to T is compared with the decision boundary between t_1 and t_2 determining which state causes a T signal output in volts (V). T encodes +V signals for t_1 and -V signals for t_2 which are received as signals in R. The received signal is compared with the decision boundary between r_1 and r_2 determining which state causes a R output message. R decodes +V signals to r_1 and -V signals to r_2. The decision boundaries, both threshold and maximum, are lower level parameters (formed by other parameters created in the implementation of T and R). These boundaries implement the relationship between each part of the t_1 - r_1 and t_2 - r_2 state-pairs and determine the operational characteristics of the signal path. A more complex communications system has more sets of transmitter and receiver state-pairs (parameters) and more complex boundaries.

Example: In the course of reading, a word appears of unknown meaning. The reader refers to a dictionary. A dictionary relates words (states) to their meanings (message). The author and reader select words from similar dictionaries (first and second comparisons). The author's and reader's dictionaries together are the state-pairs of equal words with a common meaning in each dictionary.

The state-pairs in a communications system may be created by chemical bonds (A-C, G-T in DNA), pre-existing written or spoken alphabets, pre-existing dictionaries or syntax, the specifications or standards defining a transmitter, receiver or protocol (electronic communications) or a physical implementation of a transmitter, transmission link, or receiver. Different functionalities of state-pairs are divided into layers in the Open System Interconnect model (OSI) where each reference layer provides the interface(s) used by the next layer. Layer one includes physical aspects of the interface and higher layers include successively more abstract functionality.

IV. MEASUREMENT PROCESS

With a model of a communications structure and communications process, a measurement can be defined in mathematical terms. When a measurement is understood, communication entities and their interface may be defined. Then adaptability can be measured based on the interface between the communicating entities.

A measurement is a quantified selection from an entity or process (E/P) being measured. Making a quantified selection is similar to the communications process shown in Table 1. In a measurement process there is a receiver with a set of states used for comparison with the signal from an E/P to be measured. The signal from the E/P may be generated by the E/P or applied externally to the E/P to generate a reflected signal. This signal from the E/P may be seen as from a transmitter. The receiver must have one-one states with the signals received for a practical measurement. The similarity of a measurement and a communications process supports the use of communication theory to analyze the measurement process.

N. Campbell defines a measurement (the concept) as "the assignment of numerals to represent properties" [4]. A measurement process assigns the numerals by utilizing one or more comparisons between the signal received and the states of the receiver. Each of the states of the receiver, and its associated boundary conditions, acts to quantify the measurement. Any parameter (property) of an E/P which may be quantified, e.g., weight, length, color, hardness, texture, transfer rate, capacity, spin, etc. may be a measured parameter.

The choice of the receiver, its range of states and boundary conditions actually selects and quantifies the parameter of the E/P to be measured. That is, if a length scale is used, distance is measured; if a weight scale is used, weight is measured; if a voltmeter is used, voltage is measured, etc. A measurement is not absolute; it is always relative to the parameter measured by the receiver, the states of that parameter in the receiver and the boundary conditions between states. This requires that the states of the receiver be represented in a definition of measured information.

V. DEFINING MEASURED INFORMATION

The following paragraphs develop the mathematical basis for the information contained in the measurement of a parameter (T) of an E/P. When the quantity of measured information transmitted across an interface is known then the adaptability of that interface may be quantified.

$$H(T) = -\sum_{i=1}^{i=n} p(t_i) \log p(t_i) \qquad (1)$$

$$D_t = \log n + \sum_{i=1}^{i=n} p(t_i) \log p(t_i) \qquad (2)$$

Equation (1) is Shannon's equation for entropy [1, page 50]. *p* describes the probability of *(x)*. D_t (2) is defined in T. Cover and J. Thomas as the entropy relative to *log n* [5, page 27]. This following paragraphs develop a proof that D_t represents the information contained in the measurement of a parameter (T) of an E/P.[1] A receiver (for t_i) with *n* discrete states is assumed in (2). The *n* discrete states are represented by the first term (*log n)* in (2). The entropy distribution (*H(T)*) of the measurement process is calculated by the second term. The first term is the reference applied to quantify the second term. The output from the measurement process is one or more specific states t_x, t_y, t_z. The measured information is equal to D_t.

As example, when a voltmeter (used to measure volts) with a 3 volt full scale (parameter of the voltmeter) and the minimum measurable increment (tolerance) of 0.1 V, has 30 (= n) possible states of v_i and produces a single output measurement v_x, then D_v = log 30. The greater the number of states n, the greater the information from the measurement process. The narrower the distribution of the entropy term (*H(T)*), the greater the information. A perfect measurement (zero *H(T)*) produces the maximum information, *log n*. The first term of (2) effectively includes the concept of tolerance (minimum measurable increment) in the measured information calculation.

Fig. 2 expresses (2) as a Venn diagram. Fig. 2 shows how the limit of the entropy distribution (*log n*) is related to the entropy distribution (*H(T)*). Equation (3) is Cover and Thomas' equation for Mutual Information (*MI*), the relative entropy between related entropy distributions. Replacing *H(R)* in (3) [5, page 19] with *log n* calculates the mutual information of *H(T)* and *log n* (4).

$$MI = I(T;R) = H(R) – H(R|T) \qquad (3)$$

Expand *H(R|T)=H(R)–H(T)*, then replace *H(R)* with *log n*

$$MI = \log n - (\log n – H(T)) \qquad (4)$$

[1] This definition of measured information has been proposed before. The first time may be in 1957 [6]. To the author's knowledge this paper offers the first proof of this definition.

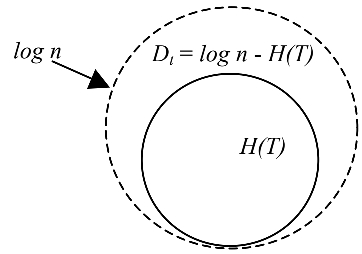

Figure 2. Venn diagram of *H(T)* and its limit.

$$MI = H(T) = I(T; \log n) \qquad (5)$$

Equation (5) shows that *H(T)* when referenced to its limit is equal to the mutual information as the *log n* terms in (4) cancel each other. Using D_t (2) provides a rigorous description of measured information without changing mutual information (MI).

In support of using *H(R|T) = H(R)–H(T)* above, a related result to (5) substitutes *H(T)* for *H(R)* in (3) and is noted as self-information [5, page 20]. Equation (5) and self-information indicate that the reference may be either *log n* or *H(T)* itself. If the reference is not *log n* or *H(T)* itself, then there are additional parameters (not T). A single parameter entropy distribution should be referenced to its limit (i.e., *log n*), as applying *H(T)* to reference *H(T)* is self-referential. The measured information related to a single parameter entropy distribution only exists in relation to a reference and the only logical reference is the limit of the entropy distribution. This provides a proof of D_t (2) as the definition of measured information.

Consider a measurement of the length of an entity using a meter scale. The meter scale is divided into 1000 increments. The entity is the same number of increments long (*x*) every time it is measured, i.e., the entropy distribution of this measurement is zero (log 1 = 0). The measured length is *x* (the data). Then the information contained in the measurement is log 1000. The quantity of measured information is the same for any accurate measurement using the same meter scale (reference).

VI. COMMUNICATIONS

Communications across an interface makes adaptability possible. This requires the six operations from Table 1 (above). These six operations occur across the interface formed by the state-pairs.

A communications system is modeled by using two overlapping Venn diagrams from Fig. 2 as shown in Fig. 3. The second Venn diagram in Fig. 3 models *H(R)* and *log* n_r and their relationship D_r. Fig. 3 is derived from Shannon's model of a communications system, where the receiver output is related to the transmitter input by a probability less than one. Fig. 3 models equation (3) with the addition of the limits of *H(T)* and *H(R)* and their associated intersection.

In Fig. 3 *log* n_t is the bound of *H(T)* and *log* n_r is the bound of *H(R)*. The intersection of log n_t and log n_r is shown as a dotted lens shape. This intersection represents the interface (*I*) made up of all the possible state-pairs of T and R. *I* limits the *MI* (intersection of the solid circles, solid lens shape) possible between T and R. *MI* represents the entropy distribution transferred from T to R. *I* is the limit of *MI* in the same manner that *log n* is the limit of *H(T)* as shown in Fig. 2. Similarly, the intersection of D_t and D_r (not delineated in Fig. 3) represents the measurement of the information that is communicated, in the same manner that D_t (2) represents the information in a measurement. When *I* (the state-pairs available for communications) is changeable, adaptability is possible.

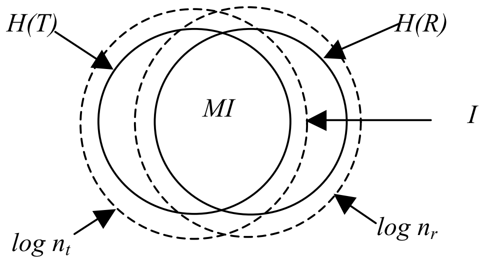

Figure 3. Venn diagram of a communication system.

By expanding on the previous example of the measurement of the length of an entity with a meter scale, a rudimentary communications process may seen. After the first measurement (with first comparison), the data (*x*) is transferred (communicated) to a second entity by applying the meter scale to a second entity and identifying where *x* appears (second comparison). The two meter scales (or one used twice) form the set of state-pairs in this communications process which pass the data (*x*) from the first entity to the second entity. This example shows that a measurement transfer system transfers data by comparing the initial entity to its reference and comparing the referenced data to the second entity.

Fig. 3 offers a similar view to the simple measurement transfer example above, where the transmitter entropy distribution (a measure of the transmitted data distribution) is compared to the common state-pairs (interface) formed by the limits of the transmitter and receiver entropy distributions. This creates the MI which, with unpaired states of the receiver, provides the received entropy distribution (a measure of the received data distribution). This view explains how communications occurs, not by an *H(T)* to *H(R)* interaction, but via the existence of a state-pairs interface.

VII. ADAPTABILITY

In a communications system, the parameters are defined by sets of state-pairs which form the interface (*I*) between two compatible E/P. The interface (*I*) allows comparisons which can support communications. Changing *I* is what allows adaptability. Using this model it is now possible to define and quantify adaptability.

A. Unpaired States

Each parameter presented across an interface consists of a number of state-pairs (*n*). However, the number of states in T may not be the same as the number of states in R for some parameters. This situation was not considered in Shannon's work, perhaps because it was seen as a design error. However, in complex communication systems such unpaired states occur when parameters, by virtue of options, special features, differing revisions or just non-selection in the transmitter or receiver, are not available or not used. For instance, telephone modems may support up to six different modulations ranging from 300 bit/s to near 56 kbit/s. Using a standardized protocol (V.8), the telephone modems identify which modulations are available at each end and select the modulation which supports the highest common data rate; the remaining modulations are unused.

Fig. 3 shows unpaired states within each dotted circle areas (*log* n_t and *log* n_r) and outside *I*. The communications structure would be more efficient if unpaired states didn't exist. Older communications systems, which tended to be single provider (e.g., telegraph and telephony), tried to avoid unpaired states to be more efficient. But the need for tolerance in older analog receivers always required the receiver states (e.g., bandwidth) be greater than the transmitter states.

Newer communications systems tend to have more and more unpaired states as communications becomes more complex and multi-vendor. This may also be seen as a greater desire for peer-to-peer systems (which will have unpaired states). Interconnected systems have become larger, multi-vendor, and include many revision levels and multiple technologies. Since memory and programming costs are very low and continue to decline, the trend towards less efficient systems (with more unpaired states) appears to be continuing.

At least two approaches have been used to avoid the effects of unpaired states: 1) the selection of other capabilities has been treated as vendor-specific and not defined (e.g., the 3G cellular IMT-2000 standards); 2) a protocol is defined to determine which of the available capabilities in the T or R should be employed in a specific situation. As example, telephone modems prior to V.32 (circa 1984) selected the modulation used based on convention and vendor-specific decision boundaries. After V.32, the identification, negotiation and selection of a specific modulation was defined by an independent protocol, V.8.

B. Defining Adaptability

Adaptability in communications systems is the process of automatically negotiating possible parameters, as it makes a system more adaptable. As defined here, adaptability requires three specific functions: identification of the capabilities available at each end, negotiation to determine the desired state-pairs (the interface), and selection of the desired state-pairs (which may require accessing software from elsewhere). These three functions are more complex versions of the three basic operations (Table 1) required for any communications: select input, compare input to reference (with adaptability mechanisms each end is compared to the other), and create output (select state-pairs). After these adaptable processes are completed, then communications of information or control can begin via the negotiated interface.

Interfaces have three major variations: fixed (state-pairs are unchangeable), flexible (state-pairs may be changed) and adaptable (state-pairs are negotiated). A mechanical plug and socket is an example of a fixed interface. Examples of flexible interfaces include: an Edison light bulb socket which supports many different types of lamps. While the mechanical aspects of the light bulb and socket are fixed, the load can be changed. A human user manually identifies and selects the specific lamp and the Edison light bulb plug/socket (the physical interface) makes this flexibility possible. A protocol example of a fixed interface that supports flexibility is the use of the Internet protocols TCP/IP as the interface with which each lower physical network or higher layer protocol is designed to interoperate. Some programmable processes also provide flexible interfaces. As example, XML (eXtensible Markup Language) provides identification and selection without negotiation.

Flexible interfaces using XML are the current state-of-the-art. Universal Plug and Play (UPnP) utilizes XML to identify and select resources between a local personal computer and peripheral devices. In this system the personal computer is "in charge" and negotiation is not necessary. Application layer negotiation to purchase music, books or programs is also widely used (e.g., Apple Store). But the negotiation of lower layer communications services (e.g., bandwidth, compression, security) where there may be a charge involved requires that the communications system itself be adaptable. As example, the compression of audio and video may be quite complex and many implementations are patented. The ability to select which form of audio or video compression is desired depends on the bandwidth available, the users' needs for audio or video quality and the users' willingness to pay. This requires a negotiation only possible with an adaptable communications system. Since the concept of adaptability is not widely understood, there are few examples of lower layer communications systems that support negotiation for the purpose of economic transactions. Yet the difficulty of equitably providing additional Internet bandwidth is well known and the even the subject of US Federal Communications Commission hearings.

A negotiation process is required to equitably support charging for lower layer communications services on the Internet. If one or more parameters are identified as proprietary (e.g., identified by a trademark), the use of such parameters would legally require the trademark owner's approval. All the other parameters used in the interface remain in the public domain. Such approval might require some form of payment to the trademark owner. If the proprietary service is valuable, implementers or users will have reason to pay the trademark owner for the use of their proprietary technology or service. Many different procedures are possible to compensate the trademark owner: charge for downloads, per implementation fees, usage fees, periodic maintenance/support fees, or simply the sales advantages of proprietary implementations or services offering improved operation over public sections of the standard.

Adaptability, which includes the means to negotiate state-pairs, is necessary to support true peer-to-peer communications. Without negotiation, one E/P must depend on the other to determine state-pairs, when unpaired states are possible. By definition, a dependent relationship cannot be peer-to-peer. Only when the two communicating E/P can change independently can they be considered peers.

C. Measuring Adaptability

Communications interfaces are layered. Adaptability or flexibility may be employed at each layer of the OSI model or partially in one or more layers. A complex interface may consists of a mix of fixed, flexible and adaptable parameters. Understanding state-pairs and the significance of the number of state-pairs allows the interface's adaptability to be measured. The adaptability of an interface may be quantified by counting the total number of state-pairs available across the interface and relating it to the total number of flexible (e.g., weighted 0.5) and adaptable (e.g., weighted 1.0) state-pairs. Arbitrarily weighting flexibility as one-half adaptability allows both approaches to changing state-pairs to be evaluated. The choice of the weighting of flexibility and adaptability as well as the weighting of specific parameters (e.g., is physical layer adaptability more or less important than higher layer adaptability) is closely related to the intended use. Equation (6) models the example of a very simple interface with three parameters b, c and d where b has n state-pairs (all fixed), c has m state-pairs (all flexible) and d has p state-pairs (all adaptable); the percentage of adaptability (A) is shown in (7).

$$a = (.5m+1p)/(n+m+p) \tag{6}$$

$$a \times 100 = A \text{ in percent} \tag{7}$$

Consider the example of the meter scale (n = 1000mm) used to transfer a measurement from one entity to another. When a meter scale is used on one side of a measurement stick and an extended yard scale (n = 16 x 39.37 inches = 629.92) is used on the other side, flexibility is introduced. Then each scale of the measuring stick is a set of fixed state pairs and the two scales are two flexible state-pairs (m=2):

1. measuring both entities with the meter scale.
2. measuring both entities with the yard scale.

The measurement of adaptability for this example is shown in (8), (9) and (10), where m=2, p=0, n_1=1000 and n_2=629.92:

$$a = (.5x2+1x0)/(1000 + 629.92+2+0) \tag{8}$$

$$a = (1)/(1631.92) \tag{9}$$

$$a \times 100 = .06\% A \tag{10}$$

D. *Implementing Adaptability*

At least two means to implement adaptability are known. Adaptability may be created by a software program (often termed agent software) that can identify, negotiate and select the state-pairs across an interface. Or an independent communications protocol may be used for the purposes of identification, negotiation and selection. When such a protocol is used only for these purposes, it is termed an *etiquette* [7]. It seems likely that other approaches to implement adaptability will be identified.

Etiquettes are already used in some communications systems, e.g., ITU V.8 for telephone modems, ITU T.30 for G3 fax, ITU G.994.1 for digital subscriber line transceivers, and IETF Session Initiation Protocol (SIP); their properties have been explored previously [7]. In the long term evolution (LTE) architecture, an etiquette would allow the service provider to negotiate the protocol that optimizes system loading or maximizes geographic coverage, or allow a user to select the protocol (and related service provider) that offers the best economic performance for that user. An etiquette also better supports troubleshooting of incompatibilities as each end can identify the available parameter sets of the other end. The use of adaptability mechanisms is a system architecture choice which significantly enhances the long term performance of programmable heterogeneous communications systems.

When systems are programmable, adaptability is possible. An etiquette transmitter presents the range of possible compatible parameters to an etiquette receiver. The etiquette receiver responds with its range of possible compatible parameters. Using heuristics local to the transmitter and receiver (e.g., largest parameter is best [pels, bits, colors, data rate, etc.]) or remote heuristics accessed by both the transmitter and the receiver (e.g., using a remote data base to determine which common parameters are to be utilized), the etiquette transmitter and receiver negotiate and select the desired interface for compatibility and follow-on communications.

Adaptability could be useful in software defined radios. A software defined radio which includes the physical layer, perhaps others, is not defined as adaptable but has the properties ─ programmable and a radio interface (non-mechanical) ─ that allow it to be adaptable.

Compatible systems have state-pairs. If there are transmitter states (at any OSI layer) that do not have related receiver states, such inconsistencies can cause "bugs." Adaptability mechanisms offer a means to negotiate and select a specific interface and reduce such bugs.

For the negotiation process of an etiquette to operate consistently, any addition to an etiquette must be a proper super-set of the previous version. As long as the etiquette is a logical single tree structure, where each branch refers to a single parameter set, no deletions are allowed and the etiquette receiver ignores anything it doesn't recognize, a correctly modified etiquette will always be backward compatible.

Following this model, an etiquette may be expanded whenever desired, independently in the transmitter and the receiver. This allows new capabilities, and the parameters in the etiquette that identify them, to be added to a communications system at any time. If both ends can support the new parameters they can be employed. If one end supports a parameter and the other end does not, it may be practical for the deficient end to download the needed software from a known Internet web site.

VIII. Conclusions

Adaptability makes it possible to automatically negotiate the rising complexity of communications, introduce new technology into communications channels at will, simplify communications troubleshooting, better support multi-mode operation, avoid identified communications channel bugs and support incentives to developers and implementers without forcing all users of public interfaces to pay private fees. The advantages of adaptability are significant. Using the approach outlined, the adaptability of current and future programmable communications systems may be measured so that users, service providers and developers may easily recognize and utilize this important functionality.

References

[1] C. E. Shannon and W. Weaver, *The Mathematical Theory of Communications*, Fig. 1 p. 34. Urbana and Chicago IL, USA: University of Illinois Press, 1963.

[2] B. Russell, *Introduction to Mathematical Philosophy*, page 15. New York: Simon and Schuster, 1971.

[3] I. Kant, *Logic* (General Doctrine of Elements, Para. 6, Logical Acts of Comparison, Reflection and Abstraction), Library of Liberal Arts, trans. R.S. Hartman and W. Schwarz. Indianapolis and New York: The Bobbs-Merrill Company, Inc., 1974.

[4] N. Campbell, *Foundations of Science*, p. 267, Dover Publications, New York, NY, 1957.

[5] T. M. Cover and J. A. Thomas, *Elements of Information Theory*, New York: John Wiley & Sons, Inc., 1991.

[6] H. Everett, III, Theory of the Universal Wave Function, page 25, 1957, published in *The Many-Worlds Interpretation of Quantum Mechanics*, edited by Bryce S. DeWitt and Neil Graham, Princeton Series in Physics, 1980.

[7] K. Krechmer, "Fundamental nature of standards: technical perspective," *IEEE Communications Magazine*, 38(6), p. 70, June, 2000. Also at http://www.csrstds.com/fundtec.html